# Fatigue reliability analysis of Cret De l'Anneau viaduct: A case study

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ABSTRACT Fatigue of reinforced concrete structures is often not considered for civil engineering structures due to the fact that dead loads of reinforced concrete structures are very high (for case of normal strength concrete) while live loads on these structures are relatively small which leads to very small stress variations during service duration of the structure. However, particularly for bridge structures with increased use of high strength concrete and increase in traffic loads this scenario is reversed and fatigue verification of these structures becomes much more important for the safety. This paper attempts to present a probabilistic framework for reliability assessment of existing bridges along with reliability-based calibration of fatigue design factors and present a case study for Cret De l'Anneau viaduct in Switzerland.

Keywords: Reliability, fatigue, reinforced concrete, bridge, calibration

## 1 INTRODUCTION

## 1.1 Early probabilistic studies on fatigue of bridges

Fatigue reliability assessment of steel components of bridges is studied in some literatures which they used for example weight in motion data to obtain reliability in orthotropic bridge deck (Yang, et al., 2016) while (Kihyon & Dan, 2010) focuses on fatigue reliability assessment of steel bridges by using probability density functions of equivalent stress range based on field monitoring data. (Saberi, et al., 2016) estimated bridge fatigue service-life using operational strain measurements. Furthermore, probabilistic reliability assessment of steel structures exposed to fatigue is studied by Krejsa (Krejsa, 2014). (Sain & Chandra Kishen, 2008) present probabilistic assessment of fatigue crack growth in steel reinforced concrete (SRC) is investigated, (Petryna, et al., 2002) proposes a time variant reliability framework along with material model for reinforced concrete, however obtained results show its inapplicability to system level of structures. Current study uses probabilistic S-N approach for fatigue reliability assessment of reinforced concrete deck of bridge where fatigue of reinforcement in tension zone investigated as fatigue of concrete in compression zone is unlikely to occur (Rocha & Brühwiler, 2012) if concrete is not suffering from any other deterioration mechanisms like frost or aggregate alkali reaction and present a case study for Cret De l'Anneau Viaduct.

## 1.2 Background and motivation

Until 1960, it was believed to be impossible to get any fatigue failure in reinforced concrete structures with

mild steel as reinforcement and with the level of permitted stresses during that time, (Mallet, June, 1991). Most of the bridges in Switzerland built during the last 50 years are reinforced concrete bridges and they typically experience more than 100 million cycles of fatigue load during design lifetime. This is especially the case for reinforced concrete decks of such bridges exposed to traffic loads during their lifetime which are not designed for fatigue (Schläfli & Brühwiler, 1998).

## 1.3 Current industry practice

Bridge engineers in the industry use Palmgren & Miner's rule of linear damage accumulation along with Wöhler curves from codes and standards, e.g. (SIA-261, 2003) for new structures and (SIA-269, 2016) for existing structures for fatigue verification of existing bridges and often with the result to replace an existing bridge or at-least the deck of the bridge.

## 1.4 Best way forward

Fatigue tests of concrete shows large scatter of fatigue lives, and use of characteristic strengths and safety factors (deterministic approach) along with code defined heavy vehicles as actions/loads, it may lead to noneconomical and non-ecological solutions, for example unnecessary replacement of bridge decks.

A best way forward could be to use reliability methods (a probabilistic approach) to obtain a more detailed assessment of the bridge and thereby a better basis for decision making. This requires a stochastic material model and a stochastic load model to be formulated, using among others monitoring of strains in the structure at critical locations. By this approach it is possible to quantify by probabilistic measures a level of damage and the remaining useful fatigue life of the structure.

This paper presents a reliability-based framework for reliability assessment with respect to fatigue failure of Crêt de l'Anneau viaduct as a case study, where the MCS department at EPFL, Lausanne, Switzerland has installed a long term monitoring system for estimating strains in the structure deck slab. As part of reliability-based framework stochastic modelling of fatigue strength of reinforcing bars along with stochastic modelling of fatigue loads will be presented. Calibration of fatigue safety factors will also be presented. The reliability value obtained will be compared with required reliability of structures as recommended by (SIA-269, 2016) Swiss Standard for Existing structures.

## 2 CRÊT DE L'ANNEAU VIADUCT AND INSTALLED MONITORING SYSTEM

## 2.1 General

Crêt de l'Anneau viaduct is an eight span composite bridge with total length of 194.8 meters, built in year 1957. Its reinforced concrete deck slab of 170 mm thickness (at mid span) is supported on two parallel steel box girders with an average height of 1.3 meters. These box girders are connected to each other by articulation, which is about four meters from support. The concrete used during construction had a cube strength of 40 MPa which now may be estimated to approximately 50 MPa with 70 years of life.  $18\varphi$  @ 500mm and  $14\varphi$  @ 100 mm reinforcement is used in the main transverse bending direction between two girders.

## 2.2 Fatigue behavior

The identified critical location of this composite bridge is the reinforced concrete slab, reference is made to (MCS, 2017). The fatigue behavior of the reinforced concrete deck slab is mainly governed by transverse bending between two girders; it contributes also to local longitudinal bending under vehicle rolling wheel loads, thus it is double bending behavior. Stress levels in the steel box girder are very low and below endurance limit for steel so the current study focuses only on reinforced concrete deck slab, and especially fatigue of the reinforcement in the tension zone and fatigue of concrete in compression zone (fatigue of concrete in compression zone is not presented as part of this paper and could be developed further in future).

## 2.3 Monitoring system installed

The MCS department at EPFL has installed eight electrical strain gauges on longitudinal and transverse reinforcement bars of two spans at halfway between articulation and support. Two more strain gauges are installed one on bottom side of top flange of box girder



Figure 1. Monitoring system installed on Crêt de l'Anneau viaduct (Strain gauges are highlighted with clouds).



Figure 2. Crêt de l'Anneau viaduct cross section.

and another on bottom side of bottom flange of box girder.

For details about monitoring system, reference is made to (MCS, 2017), Figure 1 and Figure 2.

## 3 RESULTS OF MONITORING AND STOCHASTIC LOAD MODEL

## 3.1 Strain measured and calculation of stresses

A study of influence line diagram for the bridge shows maximum stress range for live loads due to traffic can be expected at mid-span between articulation and support. At this same location strain gauges are installed at the bridge measure strain variations with a frequency of 50-100 Hz. This high frequency of strain captures all vehicles and associated peaks in responses. Along with this high frequency traffic strain measurements, the strain gauges also capture a low frequency strain change due to temperature variation and structural response due to this temperature variation. The two responses can be separated since their frequencies vary largely.

Figure 3 shows strain measured for 303 days and corresponding temperature effect.

This temperature effect can easily be removed from the total response in order to obtain the response only due to vehicles. Five to ten minutes averaging time for calculating mean temperature effect is generally



Figure 3. Effect of temperature on strain measurements.



Figure 4. Stress histogram for transverse reinforcement.

sufficient. Moving average method could be employed by using Equation 1 (National-Instruments, 2012):

$$f(y_i) = \frac{1}{2n+1} \sum_{k=i-n}^{k=i+n} y_k \quad \text{for } N-n > i > n$$
(1)

where

 $f(y_i)$  = mean temperature effect n = averaging time chosen N = total number of data points

Once the temperature effect is removed from strains, stresses in the steel reinforcement can easily be obtained.

### 3.2 Rain-flow counting and load histogram

Stress histograms are obtained by rain-flow counting for monitoring duration of 303 days. The number of cycles to failure are related only stress range for reinforcing steel which is similar to welded steel. Figure 4 shows actual stress range histogram for transverse reinforcement, as actual stresses in bridge are very low and the bridge has very high fatigue life. For illustration of reliability analysis actual histogram is scaled such that, the design equation (with characteristic values and safety factors-DFF) presented in section 4.3 is exactly fulfilled. The scaling is performed on stress range as well as number of cycles.

#### 3.3 Stochastic load model for reliability analysis

Uncertainty in fatigue load (for this case traffic load) covers different aspects and each aspect can be modeled independently. These different aspects could be e.g. measurement uncertainty in strain measurements, as these measurements are very accurate, a very small uncertainty associated with measurement is assumed and modeled as lognormal with unit mean and standard deviation of 0.05, see  $X_w$  in Table 1.

Other uncertainties could be uncertainty related to extrapolation of results to another location in structure based on measurement at a certain location (this is not considered here as strain gauges are installed at exactly same location), uncertainty related to extrapolation of available results to full year fatigue load based on 303 days observations, extrapolation of results to remaining life, which includes year-to-year variations and increase in traffic load and amount with time. Available traffic data for 303 days is extrapolated to total life of the structure by making an assumption of constant traffic over entire completed life of 70 years, this is a conservative assumption as traffic in early service duration of structure is low compared to present traffic and for future life of the structure of 50 years 1% increase in traffic volume each year is assumed. Uncertainties associated with this extrapolation is modeled as lognormal with unit mean and standard deviation of 0.10, see  $X_n$  in Table 1.

## 4 RELIABILITY FRAMEWORK

#### 4.1 General

The First Order Reliability Method (FORM) is used for reliability analysis, (Madsen, et al., 2006) & (Sørensen, 2011). An open source Matlab-based toolbox namely, the FERUM (Finite Element Reliability Using Matlab) is used for all performing FORM calculations (FERUM, July 2010).

### 4.2 Stochastic material reinforcement in reliability analysis

Deterministic Wöhler curves are recommended by various international codes e.g. (MC2010, 2013), (MC1990, 1993), (DNV OS C 502, Sept, 2012), & (EN 1992-1, 2004) etc. for verification of reinforcement fatigue. These are used as basis for establishing stochastic models together with statistical analysis of available test data, (Hansen & Heshe, 2001) for reinforcement fatigue.

For reinforcement fatigue the number of cycles required for fatigue failure can be calculated based on Wöhler curve, see Equation 2:

$$N = k \Delta \sigma^{-n}$$

or

$$\log N = \log k - m \cdot \log \Delta \sigma + \varepsilon \tag{2}$$

where  $\varepsilon$  models the uncertainty related to the SN-curve and is assumed Normal distributed mean value equal to 0 and standard deviation equal to  $\sigma_{\varepsilon}$ . The values of logk, m,  $\sigma_{\varepsilon}$  are obtained by Maximum Likelihood Method, (Sørensen & Toft, 2006), as these parameters are estimated based on limited set of the data there is an uncertainty associated with these parameters which is presented in Table 1. The use of the Maximum likelihood method provides us with the option to include run-outs. For more details about probabilistic model for fatigue strength of reinforcing bars here and associated uncertainties reference is made to (Rastayest, et al., 2018).

### 4.3 Design equation and limit state equation

The design equation for reinforcement fatigue is written see Equation 3:

$$G = 1 - \sum_{i=1}^{J} \frac{n_i T_F}{k^c} R_D \Delta \sigma_i^m = 0$$
<sup>(3)</sup>

where,

 $k^c$  is the characteristic value of k:  $logk^c = logk^{mean} - 1.64 \cdot \sigma_{\in}$   $log k^c$  corresponds to 95% quantile.  $n_i$  is number of cycles experienced by structure – for the *i*th stress block  $T_F$  is the fatigue life

 $T_F = FDF \cdot T_L$ 

*FDF* is fatigue design factor

 $T_L$  is the service life time of the structure

 $R_D$  is modelling the design parameters, here the section modulus of the deck slab

 $\Delta \sigma_{\rm i}$  is the stress range for the ith bin

Stress ranges for each bin is obtained directly by rain-flow counting of strain gauge measurements, see section 3.2.

Stress range in each bin is multiplied with the ratio of design parameters (New design parameter/ Original design parameter) this ratio is back calculated to arrive at a specific value of fatigue design factor (FDF).

This design equation can be transformed to a limit state equation by introducing stochastic variables see Equation 4:

$$g(t) = \Delta - \sum_{i=1}^{j} \frac{X_n n_i t}{10^{\epsilon} \cdot k} (X_w R_D \Delta \sigma_i)^m = 0$$
(4)

where t indicates time  $0 < t < T_L$  in years.

All other terms in the limit state equation are explained in Table 1.

#### 4.4 Calculation of reliability index

As explained in section 3.2 the actual stresses in bridge are very low and have very high fatigue life. The reliability analyses are for illustration performed using the scaled fatigue load. The cumulative (accumulated) probability of failure in time interval [0, t] is obtained by Equation 5:

$$P_F(t) = P(g(t) \le 0) \tag{5}$$

The probability of failure is estimated by FORM, see (Madsen et al., 2006). The corresponding reliability index  $\beta(t)$  is obtained by Equation 6:

$$\beta(t) = -\phi^{-1}(P_F(t)) \tag{6}$$

where,  $\Phi()$  is standardized normal distribution function.

The annual probability of failure is obtained based on cumulative probability of failure, see Equation 7:

$$\Delta P_F(t) = P_F(t) - P_F(t - \Delta t), t > 1 year$$
(7)

where  $\Delta t = 1$  year.

## 5 RESULTS AND DISCUSSION

### 5.1 General

The current age of the bridge is 70 years, and it is investigated the bridge can be used for additional 50 years, i.e. a total of 120 years. The reliability is assessed for the reinforced concrete deck slab with respect to fatigue failure of the reinforcement, as this position is often the critical location.

#### 5.2 *Code requirements for reliability*

The Swiss standard (SIA-269, 2016) provides guidelines for assessing the safety of existing structures by a probabilistic approach and presents a target reliability level in the form of reliability indices based on consequence of failure and efficiency of interventions (a unity value for coefficient of efficiency of interventions is recommended by (SIA-269, 2016), when it is not determined during the examination phase), see table 2 in Appendix B of (SIA-269, 2016).

In this paper a unity value for coefficient of efficiency of interventions is used and consequences of structural failure are assumed as serious which leads to a target annual reliability index as 4.4.

Also (EN 1990, 2002) provides some guidelines for assessment of new structures by a probabilistic approach and presents an indicative target accumulated reliability index for life time of 50 years against fatigue. It provides a range of target reliability from 1.5 to 3.8, based on degree of inspectability, repairability and damage tolerance, see table C2 in Appendix C of (EN 1990, 2002).

#### 5.3 Results of reliability analysis

Figure 5 shows the annual reliability index  $(\Delta\beta)$  as a function of FDF for different CoV value for logK. Variation of CoV values of logK show a large influence on reliability index values. To meet a target annual reliability index of 4.4 with planned design life of 120 years design engineers need to use a FDF in order of 7.5 for CoV of 0.2 for logK, while FDF is in order of 10 for CoV of 0.39 for logK.

#### Table 1. Stochastic model for Wöhler curve.

Parameter	Distribution	Mean	Standard deviation	Remark
Δ	Lognormal	1	0.30	Model uncertainty related to PM Rule*
$X_w$	Lognormal	1	0.05	Uncertainty in strain measurements
X <sub>n</sub>	Lognormal	1	<b>0.01</b> -0.1 <sup>++</sup>	Uncertainty in number of vehicles
logk	Normal	18.77	0.07	Location parameter in Wöhler curve
m	Fixed/ Deterministic	5	_	Slope of Wöhler curve fixed to 5 <sup>+</sup>
∈	Normal	0	$\sigma_{\epsilon}$	Standard deviation of error term
$\sigma_{\epsilon}$	Normal	0.39 / <b>0.20</b> **	0.06	Standard deviation of error term
$\rho_{logk,\sigma\varepsilon}$	Deterministic	0.06	-	Correlation coefficient between location and standard deviation of error

\*model uncertainty obtained by fitting lognormal distribution to test data in (CEB 1988, 1989)(Hashem 1986). +slope of Wöhler curve fixed to 5 as logk and m are highly correlated with correlation coefficient equal to 0.9997.

++Variation in reliability index as function of standard deviation of Xn values is studied.

\*\*Variation in reliability index as function of standard deviation of logK values is studied.

Values in bold indicates base values used for reliability analysis.



Figure 5. Annual reliability index as function of FDF.



Figure 6. Variation in Cumulative reliability index along service duration of structure (for a FDF of 6 and TL of 120 years).

Study of variation ranging from 1% to 10% in uncertainty associated with vehicle numbers Xn does not show noticeable variation in reliability index; same is not presented here in the form of figure.

Variation of the cumulative reliability index along the service life of the structure is presented in Figure 6 for the base case where uncertainty in vehicle number



Figure 7. Cumulative reliability index as function of FDF for TL=120 years for mean value of  $\sigma_{\varepsilon}$  equal to 0.20 and 0.39.

Xn is considered as 1% and CoV for logK is considered as 0.2.

Figure 7 shows variation of cumulative reliability indices for 120 years of design life for different values of uncertainties in location parameter logK of Wöhler curve representing reinforcement from an arbitrary delivery. It is seen that changes in uncertainty associated with logK results in large variations in reliability index however, variation in uncertainty in vehicle numbers does not show any noticeable change in reliability index. Which shows that one should focus on reducing uncertainty in logK for any case specific study to take critical decisions.

The cumulative reliability indices obtained in Figure 7 can be compared with target reliability indices indicated in (EN 1990, 2002) to obtain a range of fatigue safety factors (FDF) required to obtain the accumulated target reliability index.

### 5.4 Conclusion and future work

The reliability indices observed for the structure are larger than the acceptable range so structure can be considered as safe, also in the case where rescaling of the fatigue loads are not performed.

As the structure exhibits a very high reliability index with respect to fatigue failure of the reinforcement, the traffic load on the structure can be increased along with life extension of the structure.

Calibration of FDF is presented for different levels of reliability indices. It can be observed that bridge structures should have high FDFs to maintain same level of reliability as compared to any offshore oil and gas structures or wind turbine structures, reference is made to (Sergio & Sørensen, 2012) for wind turbines and (DNVGL RP C203, April 2016) for oil and gas structures, the reason for this could be consequence of failure and fatalities involved with such failures.

Future work could include updating the failure probability conditioning on no failure of the structure has happened for last 70 years. Which is obtained as, see Equation 8:

$$P(g(t) \le 0 | g(T_{70}) > 0), t \ge 70 \tag{8}$$

The current calibration is based on Wöhler curve Palmgren-Miner's rule and it does not include any assumption related to inspections of the structure. However, it would be interesting to include inspections of structures and calibrate FDF based on fracture mechanics this will help industry to plan reliability or risk based inspections.

Also it will be interesting to see fatigue of concrete in compression zone of bridge deck slab.

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