Sensitivity and Identifiability Study for Uncertainty Analysis of Material Model for Concrete Fatigue

Amol Mankar[a], Sima Rastayesh[a] & John Dalsgaard Sørensen[a]
[a]Aalborg University, Aalborg, Denmark
sir@civil.aau.dk

Abstract

Concrete is a widely used construction material; however, the understanding of fatigue failure in cementitious material is lacking when compared to ferrous materials. The design life of concrete structures is also evaluated using PM rule of linear damage accumulation where the fatigue strength is represented by a combination of Goodman Diagrams & Wöhler Curves. Concrete is a heterogeneous material, which is inherently full of flaws, and has a considerable scatter in fatigue test data for same test conditions. Therefore, it is desirable to introduce probabilistic concepts to ensure adequate fatigue resistance of concrete structures. This paper attempts to identify the important parameter uncertainties associated with concrete fatigue material models for uniaxial compression based on a large data set of concrete fatigue tests obtained from literature. Parameter estimation from a given dataset of experiments can be done in different ways, and in addition sensitivity and identifiability analyses can be used to search for a unique set of parameters along with their uncertainties.

Keywords: Identifiability, Sensitivity, Concrete, Fatigue, Uncertainty, Reliability

1. Introduction

The design life of concrete structures is in most design standards evaluated similar to steel structures using theory of cumulative linear damage accumulation as proposed by Palmgren, [1] and Miner, [2]. Material models for concrete fatigue are generally developed using data from testing compaigns. Waagaard in 1981 tested concrete for axial and flexural fatigue under different confining conditions in the presence of water (for offshore concrete foundations), see Det Norske Veritas (currently DNVGL), [3]. Cornelissen in 1986 tested concrete under tension fatigue at TU Delft, Netherlands [4]. Petkovic in 1990 tested high strength concrete during that time which is less than 100 MPa compresive strength for axial compression fatigue, [5]. Lohaus and others tested ultra high strength concrete with compressive strength of 180 MPa, [6]. As outcome of all this research work international codes e.g. [7], [8], [9], [10] and [11] have proposed models for fatigue of concrete which is a combination of Goodman Diagram [12] and Wöhler Curves also denoted as S-N curves. Combination of Goodman Diagram with Wöhler curve is required since fatigue of concrete is governed not only by the stress range but also the mean stresses.

All these researchers, codes and standards accepted scatter in concrete fatigue test data and proposed characteristic design curves/surfaces to be used with the partial safety factor concept. In order to obtain both reliable and cost-competitive design of reinforced and pre-stressed concrete structures, it is important that uncertainty of individual parameters are estimated and taken into account in the design process.

This can be obtained by adopting a probabilistic design philosophy where the structure is designed in order to meet a target reliability level. For this purpose uncertainty related to each parameter influencing the fatigue strength should be quantified and modeled by stochastic variable in order to estimate fatigue reliability. Thus, application of structural reliability theory could be an efficient way to adequately account for all these uncertainties while predicting fatigue lives of concrete structures, [13].

This paper presents use of statistical methods incl. sensitivity and identifiabiliy analyses for identifying a unique set of the important uncertain parameters from available dataset of experiments of concrete fatigue under axial compression, complied by [14].

The aim of this paper is to model the fatigue strength of concrete in a stochastic way in order to use it for reliability assessment of a reinforced or/and prestressed concrete component.

2. Deterministic material model

The material model is based on the latest experiments on normal-, high- and ultra-high strength concrete by [6] which is also adopted by Model Code 2010, [11]. The material model from the *fib* Model code 2010 for compression-compression loading is presented below, see equation 1, 2 and 3.

$$\log N_1 = \frac{8}{(Y-1)} \cdot \left(S_{c,max} - 1\right) \qquad (1)$$

$$\log N_2 = 8 + \frac{8 \cdot \ln(10)}{(Y-1)} \cdot \left(Y - S_{c,min}\right) \cdot \log\left(\frac{S_{c,max} - S_{c,min}}{Y - S_{c,min}}\right) \qquad (2)$$

if, $\log N_1 \leq 8$, then $\log N = \log N_1$

if, $\log N_1 > 8$, then $\log N = \log N_2$

where,

$$Y = \frac{0.45 + 1.8 \cdot S_{c,min}}{1 + 1.8 S_{c,min} - 0.3 \cdot S_{c,min}^2} \qquad (3)$$

3. Data used from literature

For the purpose of modeling the fatigue strength, experimental data that is used throughout in this paper is data from two papers [14], [15] and a thesis [16]. All these papers deal with concrete axial compression-compression fatigue tests. For the study in this paper, only high strength concrete is used and is obtained from available data filtered for strengths above 90 MPa.

4. Development of stochastic material model

Based on equation 1 and 2, all parameters with deterministic value of 8, 0.45, 1.8 and 0.3 and 1.0 are modeled as stochastic variables following normal distributions: $8 = X_2$; $1 = X_3$; $0.45 = X_4$; $1.8 = X_5$; $1.8 = X_6$; $0.3 = X_7$ and X_1 is modelled as an error term (ε) for the model equation. $X_1$is considered as normally distributed N (0, σ^2).

Equations 1, 2 and 3 are written:

$$\log N_1 = \frac{X_2}{(Y-X_3)} \cdot \left(S_{max} - X_3\right) + X_1 \qquad (4)$$

$$\log N_2 = X_2 + \frac{X_2 \cdot \ln(10)}{(Y-1)} \cdot \left(Y - S_{min}\right) \cdot \log\left(\frac{S_{max} - S_{min}}{Y - S_{min}}\right) + X_1 \qquad (5)$$

if, $\log N_1 \leq 8$, then $\log N = \log N_1$

if, $\log N_1 > 8$, then $\log N = \log N_2$

where,

$$Y = \frac{X_4 + X_5 \cdot S_{c,min}}{X_3 + X_6 S_{c,min} - X_7 \cdot S_{c,min}^2} \qquad (6)$$

All these six parameters along with the standard deviation of the error term σ_ε are estimated using the Maximum Likelihood Method (MLM). Use of the Maximum Likelihood Method provides us with the option to include runouts in the available dataset, [17]. Equation 7 shows a typical MLM function, which takes care of runouts in parameter estimation and provides a better fitting, compared over other methods e.g. least square fitting.

$$\min_{A,\sigma_\varepsilon} Likelihood\ Function(A,\sigma_\varepsilon) = \prod_{i=1}^{NF} P(N_i(A,\sigma_\varepsilon) = n_i) \cdot \prod_{i=1}^{NRunout} P(N_i(A,\sigma_\varepsilon) \geq n_i) \tag{7}$$

where A is the set of parameters, here X2 to X7, Ni = Observed failure cycles, ni = calculated (theoretical) number of cycles for failure. NF = number of observations where, fatigue failure of specimen were observed and NRunout = number of observations where, runouts (no failure) were observed.

The first term in equation 7 represents the probability for normal case of failure while second term represents a probability distribution function (cdf) for the case of runouts where number of cycles observed are greater than calculated failure cycles.

In addition, MLM provides us with uncertainty associated with each parameter, which can be directly used into a reliability analysis.

5. Parameter uncertainty and correlation

Choosing six parameters and estimating by MLM creates problem of numerous solutions, with highly uncertainty values, since most of the parameters are highly correlated. There are several ways to deal with the issues of parameter uncertainty and correlation:

1. Modify the model structure
2. Increase information content of experimental data by proper design of experiments
3. Search a parameter subset that can be reliably estimated from given data.

Solution # 1 is beyond scope of this paper and that is not the direction of research of the author, also these experiments are very costly and time consuming. Solution # 2 was attempted with use of Bootstrap methodology, [18] by generating more synthetic data, however for efficient use of Bootstrap methodology residuals should be random in nature, but for this particular model residuals were observed to follow a specific pattern and obtaining synthetic results was not possible, hence discarded. Therefore, all available information is used evaluating solution # 3.

Solution # 3 consists of performing ‘Local Sensitivity Analysis’ and ‘Identifiability Analyses’, which is explained in detail in Section 6 & 7 respectively.

6. Local Sensitivity Analysis

Local sensitivity analysis is also denoted as the one factor at a time (OAT) method. In OAT methods, each parameter/input variable is perturbed one at a time around its nominal value and resulting effect on output is measured.

Local sensitivity measures are commonly defined using first order derivative of the output, $y = f(x)$, with respect to an input parameter, x. sa represents absolute sensitivity:

$$sa = \frac{\partial x}{\partial y} \tag{8}$$

and sr represents a relative sensitivity

$$sr = \frac{\partial x}{\partial y} \cdot \frac{x^\circ}{y^\circ} \tag{9}$$

The relative sensitivity functions are non-dimensional with respect to units and very useful for comparing effect of model inputs among each other; the same is also used in identifiability analysis for identifying parameters.

The partial derivatives presented in Equation 8 can be obtained numerically by model simulations with a small positive or negative perturbation, *Δx*, of the model inputs around their nominal values, x^0. Depending on direction perturbations, the sensitivity analysis can approximated using forward, backward or central difference method. For current paper, partial derivative is performed by central difference method and shown in Equation 10.

$$\frac{\partial x}{\partial y} = \frac{f(x^0+\Delta x)-f(x^0-\Delta x)}{2\Delta x} \qquad (10)$$

A perturbation factor $\varepsilon=10^{-3}$ is used, i.e. $\Delta x = \varepsilon * x$.

All six parameters x_2 to x_7 are perturbed individually and effect of the same is observed logN and is plotted in Figure 1 against S_{min} and S_{max} values from data. It is observed that perturbation of the first three parameters gives positive and negative effects while last three parameters the effect of perturbation is very small in log N value.

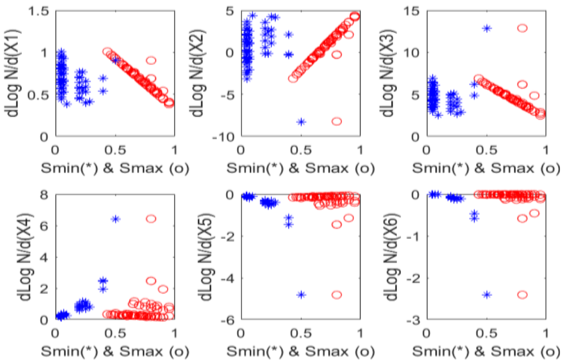

Figure 1: Effect of perturbation on Log N, plotted against S_{min} and S_{max} values

7. Identifiability Analysis

First step in the parameter estimation problem is determining which sets of parameters can be selected for estimation. The identifiability analysis is concerned with identifying those subset of parameters that, can be identified uniquely from given set of data (measurements). Uniqueness is important, in the sense that these parameters can be independently estimated accurately (with low uncertainty / variance). This also demands a low correlation between these parameters (e.g. lower than 0.5). Most of the times a lot of parameters can be used to get a better fit to the data but then the problem becomes ill conditioned. Thus; preferably an optimization can be done on the number of parameters to be estimated from a given set of data; Brun and others present a two step procedure for identifiability analysis [19], by calculating parameter significance ranking and collinearity indices, which is further explained below in detail in Section 7.1 and 7.2, respectively.

7.1. Parameter significance ranking

In this step, significance of each parameter is calculated as a non-dimensional number δ^{msqr}. Value of δ^{msqr} close form to unity indicates parameter is significant while values close to zero indicates a non-significant parameter.

$$\delta^{msqr} = \sqrt{\frac{1}{N}\sum_1^N (sr_i)} \qquad (11)$$

where sr is the relative sensitivity.

Figure 2 shows δ^{msqr} values plotted for all six parameters and it can be seen that the parameters X_2, X_3 and X_4 are significant while the others are not.

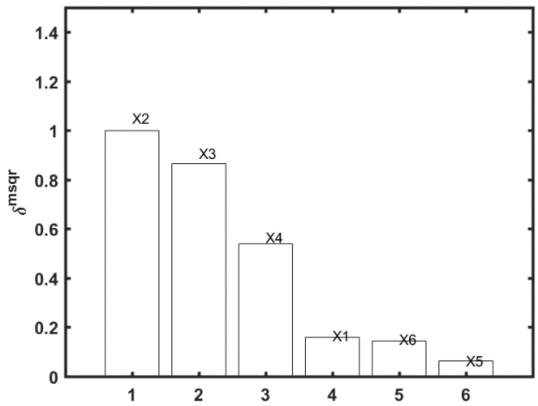

Figure 2: Parameter significance ranking

7.2. Collinearity indices

In this step, for each parameter subset (all combinations of parameter subsets which include the 2, 3, 4, 5 and 6 parameters) a collinearity index is calculated, which assess joint influence of the parameters in a given subset on the model output. A change in the model output caused by a perturbation of a parameter within the subset can be compensated in the linear approximation up to a fraction $1/\gamma_k$ by appropriate changes in the other parameters in given subset. High values of γ_k indicate that, the subset of parameters is poorly identifiable due to relations between at-least two parameters, thus totally independent vectors will have a very small value, [19], [20], [21], [22] & [23].

The collinearity index of a parameter subset k, can be calculated by Equation 12.

$$\gamma_k = \frac{1}{\sqrt{\min \lambda_k}} \qquad (12)$$

where,

$$\lambda_k = \mathrm{eigen}\left(snorm_k^T \cdot snorm_k\right)$$

$$snorm = \frac{sr}{\|sr\|}$$

is the normalized non-dimensional sensitivity function using an Euclidian norm.

λ_k are eigenvalues of normalized sensitivity matrix for parameter subset k.

Based on the collinearity indices theory, γ_k is calculated for each possible subset of six parameters. In total 57 subsets were analyzed and important subsets are identified. Identification of important subsets are done based on a criteria with a threshold of 5- 15, see [19], [20], [21], [22] & [23]. The best practice is to start with the parameter subset with the largest size (of parameters) and lowest γ_k. Results are shown in Table 1.

Table 1: Collinearity indices for each subset

k	Size of k	Parameter combination						γ_k
1	6	X2	X3	X4	X5	X6	X7	852.7
2	5	X3	X4	X5	X6	X7		118.7
3	5	X2	X3	X5	X6	X7		852.5
4	5	X2	X3	X4	X5	X6		126.8
5	5	X2	X3	X4	X6	X7		106.7
6	5	X2	X3	X4	X5	X7		93.45
7	5	X2	X3	X4	X5	X6		77.52
8	4	X2	X3	X4	X5			25.41
9	4	X2	X3	X4	X6			33.58
10	4	X2	X3	X4	X7			52.58
11	4	X2	X3	X5	X6			11.89
12	4	X2	X3	X5	X7			5.69
13	4	X2	X3	X6	X7			10.47
14	4	X2	X4	X5	X6			76.43
15	4	X2	X4	X5	X7			92.36
16	4	X2	X4	X6	X7			105.9
17	4	X2	X5	X6	X7			125.3
18	4	X3	X4	X5	X6			11.58
19	4	X3	X4	X5	X7			5.48
20	4	X3	X4	X6	X7			10.02
21	4	X3	X5	X6	X7			50.51
22	4	X4	X5	X6	X7			117.1
23	3	X2	X3	X4				13.54
24	3	X2	X3	X5				2.12
25	3	X2	X3	X6				2.08
26	3	X2	X3	X7				2
27	3	X2	X4	X5				22.71
28	3	X2	X4	X6				30.37
29	3	X2	X4	X7				48.52
30	3	X2	X5	X6				11.02
31	3	X2	X5	X7				5.34
32	3	X2	X6	X7				9.75
33	3	X3	X4	X5				2.25
34	3	X3	X4	X6				2.2
35	3	X3	X4	X7				2.08
36	3	X3	X5	X6				11.11
37	3	X3	X5	X7				4.73
38	3	X3	X6	X7				7.7
39	3	X4	X5	X6				10.49
40	3	X4	X5	X7				5.07
41	3	X4	X6	X7				9.2
42	3	X5	X6	X7				47.49
43	2	X6	X7					5.17
44	2	X5	X7					3.27
45	2	X5	X6					8.56
46	2	X4	X7					1.36
47	2	X4	X6					1.68

48	2	X4	X5	1.87
49	2	X3	X7	1.05
50	2	X3	X6	1.19
51	2	X3	X5	1.29
52	2	X3	X4	1.74
53	2	X2	X7	1.25
54	2	X2	X6	1.51
55	2	X2	X5	1.68
56	2	X2	X4	12.61
57	2	X2	X3	1.83

8. Results and Conclusion

From Table 1 it can be observed that the parameter combination of X_2, X_3 & X_4 can be considered as the best combination of important parameters that can be estimated out of the given dataset (highlighted in red), as collinearity index for these parameters is 13.54, which is in required range of 10 – 15, based on [19], [20], [21], [22] & [23]. Figure 2 also exhibits the same subset of parameters since the parameter significance ranking is also higher for these parameters among six.

Based on the results of sensitivity and identifiability analysis, the parameter subset is chosen and can be estimated by the Maximum Likelihood Method (MLM). The material model can be updated and fitted to the data with mean values of estimated parameters. This new fit would be a better fit than model code fit to the data. Presentations of these results are outside the scope of this paper and will be presented in a subsequent paper.

Acknowledgements

The project INFRASTAR (infrastar.eu) has received funding from the European Union’s Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 676139. The grant is gratefully acknowledged.

References

[1] A. Palmgren, "Die lebensdauer von kugellagern (The life of ball bearings)," *Zeitschrift Des Vereins Deutscher Ingenieure*, vol. 68, no. 14, pp. 339-341, 1924.

[2] M. A. Miner, "Cumulative damage in fatigue," *American Society of Mechanical Engineers - Journal of Applied Mechanics*, vol. 12, no. 3, pp. 159-164, 1945.

[3] K. Waagaard, "Fatigue strength of offshore concrete structures," Det Norske Veritas, Hovik, Norway, 1981.

[4] H. A. W. Cornelissen, "Fatigue failure of concrete in tension," Stevinweg 4, P.O. Box 5048, 2600 GA Delft, The Netherlands, 1986.

[5] G. Petkovic, H. Stemland and S. Rosseland, "Fatigue of High strength Concrete," *American Concrete Institute (International Symposium)*, pp. 505-525, 1992.

[6] L. Lohaus, N. Oneschkow and M. Wefer, "Design Model for fatigue behaviour of normal-strength, high-strength and ultra-high-strength concrete," *Structural Concrete*, 2012.

[7] DNV OS C 502, "DNV OS C 502, Offshore concrete structures," DNVGL, Høvik, Sept, 2012.

[8] NEN 6723, "Regulations for concrete - Bridges - Structural requirements and calculation methods," Netherlands, 2009.

[9] EN 1990, "EN 1990 Basis of structural Design," European committee for standardisation, Brussels, 2002.

[10] MC1990, "FIB Model Code for concrete structures 1990," Berlin , 1993.

[11] MC2010, "FIB Model Code for concrete structures 2010," Ernst & Sohn, Berlin Germany, 2013.

[12] J. Goodman, "Mechanics Applied to Engineering," *Longman, Green & Company*, 1899.

[13] H. O. Byung, "Fatigue analysis of plain concrete in flexure," *Structural Engineering*, vol. 112, no. 2, pp. 273-288, 1986.

[14] E. Lantsought, "Fatigue of Concrete under compression Database and proposal for high strength concrete," TU-Delft, Delft, 2014.

[15] E. V. Sørensen, "Fatigue life of high performance grout in dry and wet environment for wind turbine grouted connections," AAU, Aalborg, 2011.

[16] M. Thiele, "Experimentelle Untersuchung und Analyse der Schädigungsevolution in Beton unter hochzyklischen Ermüdungsbeanspruchungen," BAM, Berlin, 2016.

[17] J. D. Sørensen and H. S. Toft, "Probabilistic Design of Wind Turbines," *Energies*, vol. 3(2), pp. 241-257, 2006.

[18] B. Efron, "Bootstrap methods: another look at the jackknife," *The annals of Statistics* , 1979.

[19] R. Brun, M. Kühni, H. Siegrist, W. Gujer and P. Reichert, "Practical identifiability of ASM2d parameters—systematic selection and tuning of parameter subsets," *Water Research*, vol. 36, no. 16, pp. 4113-4127, 2002.

[20] G. Sin and P. A. Vanrolleghem, "Extensions to modeling aerobic carbon degradation using combined respirometric–titrimetric measurements in view of activated sludge model calibration," *Water*, vol. 41, no. 15, pp. 3345-3358, 2007.

[21] G. Sin, A. S. Meyer and K. V. Gernaey, "Assessing reliability of cellulose hydrolysis models to support biofuel process design—identifiability and uncertainty analysis," *Computers & chemical*, vol. 34, no. 9, pp. 1385-1392, 2010.

[22] G. Sin, S. M. Anne and V. G. Krist, "Assessing reliability of cellulose hydrolysis models to support biofuel process design—Identifiability and uncertainty analysis," *Computers & Chemical Engineering* , vol. 34, no. 9, pp. 1385-1392, 2010.

[23] G. SIn and V. G. Krist, "Data handling and parameter estimation" In "Experimental Methods In Wastewater Treatment", London: IWA Publishing, 2016.

[24] C. Zanuy, L. Albajar and P. d. l. Feunte, "Sectional Analysis of Concrete Structures under Fatigue Loading," *ACI Structural Journal*, pp. 667-677, September-October 2009.